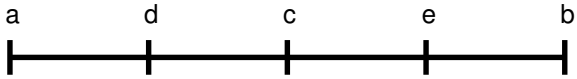
Homework #6
APM 111: Introduction to numerical methods
due April 11, 2006

1. Simpson's rule: show that given an interval $[a,b]$ as in the following figure:



the expression for Simpson's rule applied to $[a,c]$ and $[c,b]$ separately and combined is

$$S_2 = \frac{h}{12}(f(a)+4f(d)+2f(c)+4f(e)+f(b))$$

with $h = b-a$.

2. Show that Simpson's rule is fourth order. That is, show that it integrates exactly cubic polynomials, but not quartics.

3. Number of evaluations as function of tolerance: use the `quadtx.m` from the demo library of the textbook "Numerical computing with Matlab" (link on extended syllabus) to find the number of required iterations to solve for the integral $\pi = \int_{-1}^{1} \frac{2}{1+x^2} dx$ from 0 to 1, to within a tolerance of 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}. Example:

```
[Q,cnt]=quadgui(inline('2/(1+x^2)'),-1,1,1.e-6)
```

Plot the number of evaluations as function of the tolerance on a log-linear scale or a log-log scale (try different possibilities to see which makes the most sense) to see the underlying behavior. Plot also the error in the integral (error is simply `Q`$-\pi$) as function of the tolerance. Discuss your results.

4. Being careful with integration routines, and learning another way to specify the integrand in Matlab:

   (a) Evaluate the integral $\int_0^{2\pi} \cos^2 x\, dx$ analytically.

   (b) Create a Matlab M-file `cossquare.m` that contains the following two lines

```
function f=cossquare(x)
f=cos(x)^2
```

   and which evaluates $\cos^2(x)$ given an argument x (to learn about M-files, see the textbook, section 6.4). Use the Matlab quadrature demo routine to evaluate this integral.

```
quadtx(@cossquare,0,4*pi)
```

   What is the result? Use `quadgui` to see why `quadtx` and `quadgui` fail and explain what you find.