Homework #5
APM 111: Introduction to numerical methods
due Mar 23, 2006

1. Least squares for fitting a non-straight curve: formulate the problem of fitting data with a quadratic polynomial $y = ax^2 + bx + c$. Solve analytically for the coefficient a, b, c in terms of N given data points (x_i, y_i), $i = 1, ..., N$, $N > 3$. (See Burden and Faires for the analytic solution in the case of a straight line).

2. Fit a quadratic line through the log of the data points for the Oil prices, since 1910 only, (you may use your solution from the previous section, or use Matlab's least square solver `polyfit` which uses the backslash operator). The data are in the file `Oil_HW.dat` in the supporting materials directory, least squares sub directory, under the course home page. Plot the data and the fitted straight line.

3. Produce a random time series of length N, zero mean and unit variance using Matlab's `rand` command (Matlab's `rand` command produces random numbers from 0 to 1. You need to remove the average of these, and multiply by a scaling factor to get random numbers with zero mean and the desired variance); calculate $\overline{\sigma_i \sigma_{i+1}}$ for $N = 10, 100, 1000, 10000$. (Reminder: $\overline{P_i} = (1/N)\sum_{i=1}^{N} P_i$). Explain the results.

4. Markov processes:

   (a) Produce and plot a series of N numbers θ_i from a Markov process with $R = .85$ and $\sigma = 20$ (again, Matlab's `rand` command produces random numbers from 0 to 1; you need to remove the average of these, and multiply by a scaling factor to get the white noise σ_i with zero mean and variance $\sigma^2 = 20^2$). Make sure you use a large enough number of points, N, to see the full typical behavior of the Markov process.

   (b) Using $R = .95$ and $\sigma = 2$, generate a random time series with $N = 10000$. Use Matlab to estimate $\overline{\theta_i \theta_{i+j}}$ for $j = [0 : 1 : 100]$. Plot your estimate of $\overline{\theta_i \theta_{i+j}}$ versus j using first the $N = 10,000$ time series, and graphically compare your estimate with the expected theoretical result derived in class. Discuss your results.

   (c) **Optional extra credit (easy):** Plot the absolute value of your estimate of the autocorrelation function $\overline{\theta_i \theta_{i+j}}$ using `semilogy`. Explain what you see.

5. **Optional challenge problem:** (possibly a bit more of a challenge than in previous HW) Write down the Householder transformation that zeros (only) the elements $a_{k,2}$ and $a_{k+1,2}$ of a matrix $A = a_{i,j}$. That is, find the elements of the vector w such that $P = I - 2ww^T$ and $A^{(2)} = PAP$ has the above elements zeroed. See Burden and Faires' discussion of the Householder transformation (in a different context than was discussed in class) for some hints.