Homework #4
APM 111: Introduction to numerical methods
due Mar 16, 2006

1. Write a Matlab program that finds the zero of $f(x) = e^x + x$ using the secant method. Start with a reasonable initial guess and iterate until convergence, printing the iterates for x_n at every iteration. The convergence criteria should be that the distance between iterates is equal or smaller than 10^{-5}. Print the iterates with an accuracy of 15 digits using the "format long" Matlab command. Please submit the program .m file (to Mike) as well as its output.

2. Calculate the fractal dimension of the object obtained as follows: start with the [0,1] interval. at the first iteration, eliminate the middle half, leaving a quarter on each side. at the next iteration, eliminate the middle half of each of the remaining sub intervals, etc.

Figure 1: First few stages of the fractal in Question #2

3. Chaos and sensitivity to initial conditions in the logistic map (can use Matlab program from supporting material directory):

   (a) Plot x_n as function of n for solutions with a period of 2, 4, 8 and 16. Change the logistic map coefficient to find a chaotic solution and plot it as a function of n as well. Be sure to label your plots' axes (Matlab's 'xlabel', 'ylabel', 'title' commands), and the individual curves, if you plot more than one on the same graph (Matlab's 'legend' command).

   (b) **Optional challenge problem:** (i) When the map is in the chaotic regime, start from some initial conditions and plot the solution as function of n. Then start with another initial condition that is different by 0.01. Plot the iterates starting from both initial conditions on the same plot. (ii) Plot the difference between x_n for the two runs, as function of n. do so using a log-linear plot. That is, the y axis being logarithmic and the x axis linear. Discuss and interpret the results for the difference between the runs as function of n.