Homework #3
APM 111: Introduction to numerical methods
due Mar 9, 2006

1. Use Vandermonde matrix and full polynomial interpolation to interpolate the function $cos^2 x$ in the domain $[0, 2\pi]$. Repeat for the function $cos(x)$. Follow the following steps:
   (a) Calculate and write $n = 20$ equally spaced data points for the function to be interpolated, including the end points.
   (b) Calculate the Vandermonde matrix, and write its conditioning number based on the 2-norm.
   (c) Solve for the polynomial coefficients, plot the original data points, the original function to be interpolated and the full polynomial interpolation.
   (d) Plot also the piecewise cubic interpolation for the two data sets using the built in the Matlab function.
   (e) Discuss your results for the two functions and try to explain any differences between them and between the full polynomial and piecewise cubic interpolations.
   (f) **Optional challenge problem:** calculate and plot the full polynomial interpolation for theses two functions using knots (sampling locations) at the location of the Chebyshev polynomial zeros.

2. Cubic spline interpolation: derive the matrix equation for the slopes at the "knots", d_k, when the interpolated function is known to be periodic in the domain $[x_0, x_n]$. Use the following boundary conditions in this case $P'(x_0) = P'(x_n)$ and $P''(x_0) = P''(x_n)$, writing them explicitly in terms of x_k, y_k, δ_k. You may assume $h_k = x_{k+1} - x_k = h = constant$. Explain why these boundary conditions make sense and why two different boundary conditions are needed (in terms of the number of unknown vs the number of equations).

3. **Optional challenge problem:** find and plot on a linear-log scale (using Matlab's `semilogy` function) the conditioning number of the Vandermonde matrix for $n = 4:1:20$ on the interval $[-1, 1]$ for uniform sampling, and for sampling at the zeros of the Chebyshev polynomial of the first kind. See wikipedia links under the supporting material directory for the formula for the zero locations. Discuss your results. This is actually quite an easy problem.