Homework #2
APM 111: Introduction to numerical methods
due Tuesday, Feb. 28, 2006

1. Use the normalized power method to solve for first eigenvector of the following matrix

$$A = \begin{pmatrix} 10 & -7 & 0 \\ -39 & 2.099 & 6 \\ 5 & -1 & 5 \end{pmatrix}.$$

Let $\vec{e}^{(k)}$ be the kth iteration of the power method for the solution of the eigenvector belonging to the largest eigenvalue, $\vec{e}_1$. Use $\vec{e}_1^{(0)} = (3,2,1)$

   (a) Write down $\vec{e}_1^{(k)}$ for $k = 1,2,3,5,10,20$.

   (b) Plot $||\vec{e}_1 - \vec{e}_1^{(k)}||_2$ as function of k for $k = 1,...,20$.

   (c) Use $\vec{e}_1^{(k)}$ eigenvector to calculate the approximate largest eigenvalue $\lambda_1^{(k)}$ using $\lambda_1^{(k)} = (\vec{e}_1^{(k)})^T(A\vec{e}_1^{(k)})$. Explain why this is an approximation for the largest eigenvalue. Calculate the largest eigenvalue λ_1 using `eigs(A)` in Matlab, and plot $\lambda_1^{(k)} - \lambda_1$ as function of k for $k = 1,...,20$.

2. Create a random matrix using `A=rand(n)` and plot the conditioning number of this matrix as function of n for $n = 1,...,20$ using the three norms discussed in class (can use `cond(A,1)`, `cond(A,2)`, `cond(A,inf)`).

3. **Optional extra credit:**[1] Write a Matlab solver for a band-5 matrix (non zero elements only on the diagonal and on the two lines above and two below the diagonal). Assume that no pivoting is needed. You may want to start from the tridiagonal solver given in CM's book.

   (a) Demonstrate how this solver works on a 10×10 matrix in which the non zero elements are random numbers between -1 and 1, except for the diagonal which has random numbers from 3 to 4.

   (b) What is such a matrix called?

   (c) Compare your results to that obtained by the backslash operator in Matlab.

[1] What's the point of optional extra credit problems: apart from the fun of doing them, they will count instead of homework problems in which you may have missed an answer...