Geometric interpretation of stokes theorem, from Wikipedia $\int_S \hat{n}\cdot\nabla\times\vec{v}\,dA = \oint_C \vec{v}\cdot d\vec{R}$

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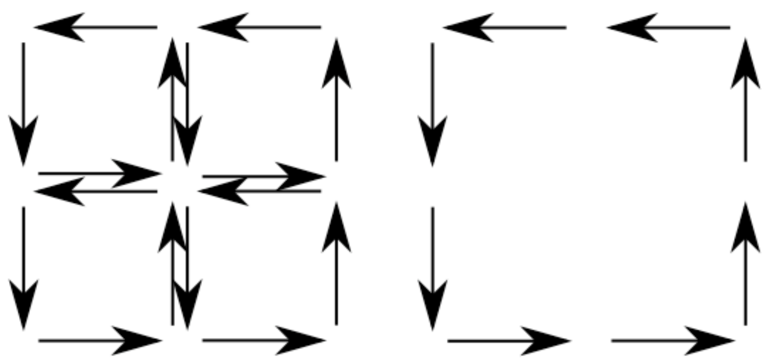
Remember that curl is related to the twisting motion of a vector field (e.g. fluid velocity), and for a solid body rotation in 2d we have $\nabla\times\vec{v} = 2\omega$ where ω is the angular velocity of the rotation.



Consider the following 2d domain. In the l.h.s. of the sketch one sees four small, identically oriented tiles these represent the twisting motion as measured on a small scale by the curl of the velocity field on the domain (velocity field itself is not drawn).

Suppose now that we integrate over the vorticity of the entire domain, which would correspond to adding together the vorticity of the four tiles. The ”interior paths” shown for adjacent tiles run in opposite directions; their contributions to the path integral thus compensate each other pairwise. As a consequence, only the contribution from the outside edge curve remains. This edge contribution is simply the line integral on the rhs of the stokes theorem.