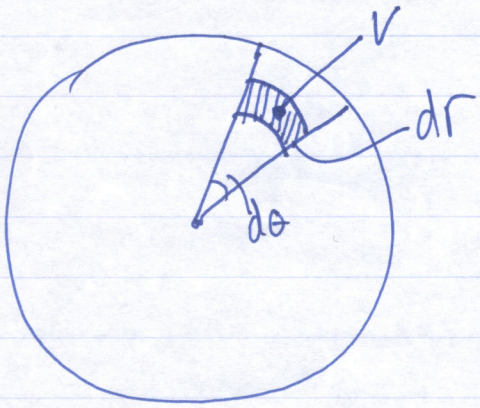
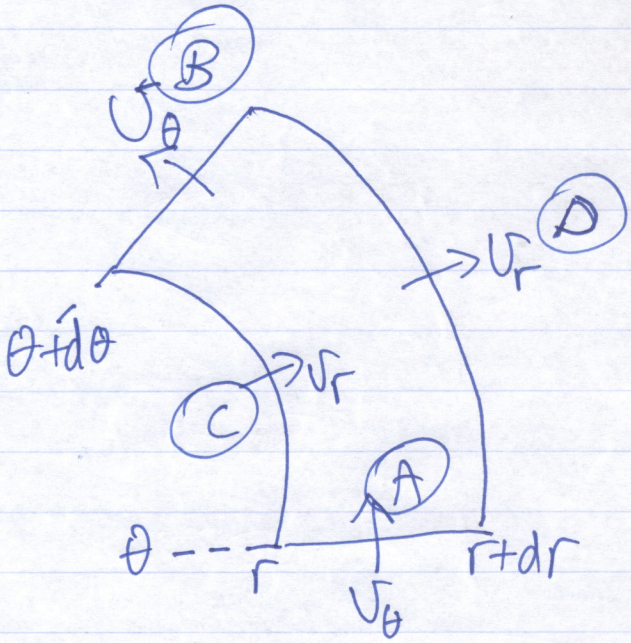
Divergence in polar coordinates

$$\vec{\nabla}\cdot\vec{v} \equiv \lim_{V\to 0}\left(\frac{\text{outflow}}{V}\right),$$





area: $(r\cdot d\theta)dr$

$\frac{1}{\text{area}}\cdot$ outflow: $(B-A+D-C)\frac{1}{r\,dr\,d\theta}$

$$= \frac{\left(v_\theta dr\big|_{\theta+d\theta} - v_\theta dr\big|_{\theta}\right)}{r\,dr\,d\theta} + \frac{v_r(r\,d\theta)\big|_{r+dr} - (v_r r\,d\theta)\big|_{r}}{r\,dr\,d\theta}$$

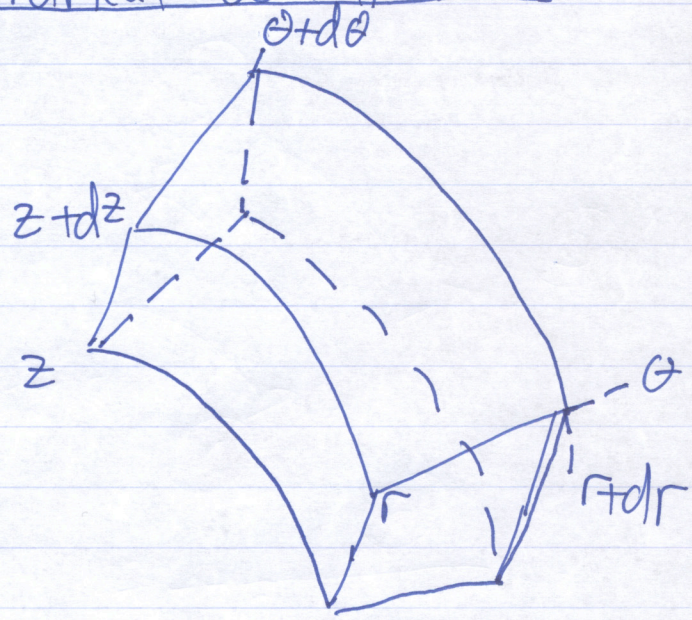
$$= \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{1}{r}\frac{\partial}{\partial r}(r v_r)$$

* mention alternative approach of using

$$\vec{\nabla}\cdot\vec{u} = \left(\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{e}_z\frac{\partial}{\partial z}\right)\cdot\left(\hat{e}_r u_r + \hat{e}_\theta u_\theta + \hat{e}_z u_z\right)$$

& the need to know $\frac{\partial\hat{e}_r}{\partial\theta}$ etc.

Divergence in cylindrical coordinates



now need to add outflow from upper & lower faces,

& volume is

$r\,d\theta\,dr\,dz$.

face areas are: sides: $dr\,dz$

front & Back: $r\,d\theta\,dz$

top & bottom: $r\,d\theta\,dr$

$$\Rightarrow \vec{\nabla}\cdot\vec{v} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial r}\left(r v_r\right) + \frac{\partial v_z}{\partial z}$$