Curl: application to the management of fisheries

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1 Coriolis force and Ekman transport

The Coriolis force acts on moving objects on earth and is a result of the earth rotation. In the northern hemisphere, it always acts to the right of the velocity (to the left in the southern hemisphere), and the force is proportional to the velocity. As a result, the velocity at the surface of the ocean is to the right of the wind blowing at the surface, rather than being in the direction of the applied wind force! Denoting the wind components in the (east, north) directions as $\boldsymbol{\tau} = (\tau^{(x)}, \tau^{(y)})$, and the two horizontal velocity components as (u, v). The force balance may then be written as

$$\begin{aligned} -fv &= \tau^{(x)} & (1) \\ fu &= \tau^{(y)}, & (2) \end{aligned}$$

or, dividing by the Coriolis parameter $f \approx 2\pi/(24\ hours) \approx 10^{-4} sec^{-1}$,

$$\begin{aligned} v &= -\tau^{(x)}/f & (3) \\ u &= \tau^{(y)}/f. & (4) \end{aligned}$$

We showed in class that the mass conservation equation for an incompressible fluid such as ocean water is given by

$$div\,\vec{u} = u_x + v_y + w_z = 0.$$

Since the horizontal ocean currents do not depend on depth in the upper 50m or so of the ocean (being well mixed), we can integrate this equation in the vertical, using also the fact that the vertical velocity vanishes at the surface of the ocean, to find

$$\int_{-50m}^{0} dz(u_x + v_y) = (50m)(u_x + v_y) = \int_{-50m}^{0} -w_z = -(w(0) - w(-50m)),$$

so that

$$(50m)(u_x + v_y) = w(-50m).$$

Using the force balance above, we can write

$$u_x + v_y = \frac{1}{f}\left(\frac{\partial}{\partial x}\tau^{(y)} - \frac{\partial}{\partial y}\tau^{(x)}\right)$$

which, combined with the previous equation implies that the upwelling velocity is given by the curl of the wind,

$$w(-50m) = \frac{50m}{f}\left(\frac{\partial}{\partial x}\tau^{(y)} - \frac{\partial}{\partial y}\tau^{(x)}\right) = \frac{50m}{f}\nabla\times\boldsymbol{\tau}.$$

The simplest case to consider is a northward, along-cost wind near a coast that's oriented in the north-south direction, and which vanishes away from the coast. see slide.