why the obsession with S-L problems?

consider the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
with initial conditions (string position when plucked & before being released):
$u(t=0, x) = f(x)$, & boundary conditions:
$u(t, x=0) = u(t, x=L) = 0$.
attempt a solution of the form
$u(t,x) = T(t) \cdot X(x)$. subst in eq'n:

$$\frac{\partial^2 T}{\partial t^2} \cdot X(x) = c^2 \cdot T(t) \cdot \frac{\partial^2 X}{\partial x^2}$$. divide by $X \cdot T \cdot c^2$

$$\Rightarrow \frac{1}{c^2} \frac{1}{T(t)} \frac{\partial^2 T}{\partial t^2} = \frac{1}{X(x)} \frac{\partial^2 X}{\partial x^2} = \lambda = \text{const.}$$

$$\Rightarrow \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \lambda \Rightarrow X'' = \lambda X, \; X(0) = X(L) = 0$$

$\Rightarrow$ a S-L problem!
need to find all possible λ_n, ϕ_n,
completness of ϕ_n is needed when we try to satisfy the i.c. $u(t=0, x) = f(x)$ by expanding $f(x)$ in $\phi_n(x)$.