Available online at www.sciencedirect.com
SCIENCE DIRECT®

Physics of the Earth and Planetary Interiors 140 (2003) 127–143

PHYSICS OF THE EARTH AND PLANETARY INTERIORS

www.elsevier.com/locate/pepi

Thermal and magnetic evolution of the Earth's core

Stéphane Labrosse*

Institut de Physique du Globe de Paris, 4, place Jussieu, 75252 Paris Cedex 05, France

Accepted 11 July 2003

Abstract

The magnetic field of the Earth is generated by convection in the liquid-core and the energy necessary for this process comes from the cooling of the core which provide several buoyancy sources. The thermodynamics of this system is used to relate the Ohmic dissipation in the core to all energy sources and to model the thermal evolution of the core. If the same dissipation is maintained just before the onset of inner-core crystallization, and the associated compositional convection, as at present, a much larger heat flow at the core mantle boundary (CMB) is necessary which, if extrapolated backward, may require a very high initial temperature. Two solutions to that problem are studied: either the Ohmic dissipation was smaller then, which could be maintained with the same heat flow as at present or an important radioactivity is present in the core. The presence of radioactivity in the core makes the inner core only a few hundred million years (Ma) older than non-radioactive cases with the same dissipation, because the low efficiency of radioactive heating requires a much larger heat flow at the core mantle boundary. Although the age of the inner core is controlled by the heat flow at the CMB, the Ohmic dissipation to be maintained is the constraint that makes it low.

Keywords: Thermal evolution; Magnetic evolution; Earth's core; Inner core

1. Introduction

The thermal evolution of the Earth is at the origin of all its dynamics. In particular, the cooling of the Earth's core provides several buoyancy sources that maintain convection in its outer liquid part and generates the Earth's magnetic field by dynamo action. This process must satisfy some thermodynamical constraints and this gives us a chance to estimate the cooling rate of the core.

A global entropy balance can be written for the convective core relating the Ohmic dissipation to all existing energy sources which are both thermal and compositional when an inner core is present and crystallizing and only thermal before the existence of the inner core. This balance shows that the compositional energy is more efficient than any thermal source in maintaining a given dissipation and that the efficiency of a given thermal source depends on the temperature contrast between the heat source and the core mantle boundary (CMB). When the inner core is present, its growth rate can be computed if the Ohmic dissipation is known by use of the entropy equation. Before the existence of the inner core, the cooling rate of the core is computed in that way. The global energy balance of the core can then be used to compute the heat flow out of the core that is necessary to maintain a given Ohmic dissipation in the core.

These thermodynamic relations and their parameterization in terms of inner-core radius or temperature at the center of the core are given in Section 2 and

* Fax: +33-1-44-27-49-32.
E-mail address: labrosse@ipgp.jussieu.fr (S. Labrosse).

0031-9201/$ – see front matter
doi:10.1016/j.pepi.2003.07.006

the rest of the paper is devoted to the analysis of several cooling scenarios based on these relations. Particular attention is given to the effect of the value of the Ohmic dissipation both at present and just before the inner core started to crystallize. Another set of models is produced to study the effect of radioactivity in the core.

The present Ohmic dissipation in the core is poorly known (Section 3) and even more so for the past history of the Earth. Ideally, one would like to infer it from the paleomagnetic record but this seems out of reach until a better understanding of the link between the dipole variability and the dissipation is obtained (Labrosse and Macouin, 2003). Previous studies (Buffett, 2002; Gubbins et al., 2003a) have tried to make a thermal evolution model maintaining the present dissipation before the inner core as well as at present, a very difficult task because, as will be shown below (Section 4.2), the much smaller efficiency of thermal convection compared to compositional convection would require a much larger heat flow at the CMB at that period, which may imply a very large temperature at the CMB at the origin of the Earth, if extrapolated backward in time. This problem can be solved either by adding some radioactivity in the core (Section 4.4) or by simply assuming a smaller dissipation prior to inner-core formation (Section 4.3).

Beside helping for the initial temperature problem, the eventuality of the radioactivity in the core does not really make the inner core much older. Because radioactivity is the least efficient of all energy sources, a much larger heat flow at the CMB is needed in presence of radioactivity to maintain the same Ohmic dissipation. Although the age of the inner core is controlled by the heat flow at the CMB, the Ohmic dissipation to be maintained in the core places limits on its value, through the efficiency calculations. The presence of radioactive elements in the core is then allowed by thermodynamics but certainly not a necessity.

2. Thermodynamics of the core

2.1. Mean state of the core

The thermal evolution of the core is controlled by mantle convection that imposes the total heat loss of the core. The time scale of this evolution is then of the order of tens of millions of years, much larger than the time scales relevant to the dynamics of the fluid core. It means that this dynamics can be averaged out while dealing with the thermal evolution (Braginsky and Roberts, 1995) and enters in the problem only in maintaining the core close to the mean state, that is usually assumed: hydrostatic, isentropic and well mixed (uniform mass fraction of light elements). It is often more convenient to use the temperature instead of the entropy as a state variable and the condition of uniform composition and isentropy imply that the temperature follows an adiabat (e.g. Braginsky and Roberts, 1995; Labrosse et al., 1997). The pressure P, mass fraction of light elements ξ and temperature T_{ad} in the mean state of the outer core are solution of:

$$\frac{\partial P}{\partial r} = -\rho g, \tag{1}$$

$$\frac{\partial \xi}{\partial r} = 0, \tag{2}$$

$$\frac{\partial T_{\mathrm{ad}}}{\partial r} = -\frac{\alpha g T_{\mathrm{ad}}}{C_P}, \tag{3}$$

with ρ the density, g the acceleration due to gravity, α the coefficients of thermal expansion and C_P the heat capacity by unit mass. To solve the problem, an equation of state is needed, and we use the logarithmic equation of state proposed by Poirier and Tarantola (1998):

$$P = K_0 \frac{\rho}{\rho_0} \ln \frac{\rho}{\rho_0}, \tag{4}$$

with ρ_0 and K_0 the density and incompressibility at zero pressure, respectively. In addition, the gravity profile is related to the density profile by

$$g(r) = \frac{4\pi G}{r^2} \int_0^r u^2 \rho(u)\, \mathrm{d}u. \tag{5}$$

This set of non-linearly coupled equations cannot be solved exactly and two approaches are possible. One can obtain a fit for one profile from a radial seismological model of the core, PREM (Dziewonski and Anderson, 1981) for instance, and get the other profiles from Eqs. (1)–(5)). This approach is very efficient for the present Earth but impractical for earlier periods. Another approach, used here, was proposed by Labrosse et al. (2001) and need not be replicated here.[1] The expression for the density and gravity profiles obtained are

$$\rho = \rho_c \exp\left[-\frac{r^2}{L_\rho^2} + O\left(\frac{r^4}{L_\rho^4}\right)\right], \quad (6)$$

$$g(r) = \frac{4\pi}{3} G\rho_c r\left[1 - \frac{3r^2}{5L_\rho^2} + O\left(\frac{r^4}{L_\rho^4}\right)\right], \quad (7)$$

with ρ_c the density at the center and L_ρ a length scale for the compression given by:

$$L_\rho = \sqrt{\frac{3K_0}{2\pi G\rho_0\rho_c}\left(\ln\frac{\rho_c}{\rho_0} + 1\right)} = 7400 \pm 150\,\text{km}, \quad (8)$$

where the numerical value is obtained from a fit to the PREM densities.

These profiles are obtained without taking into account the density change across the inner-core boundary (ICB) coming from both the phase and composition change. This effect can be included (Labrosse et al., 2001) when needed.

The adiabatic temperature profile can then be integrated directly from Eq. (3)

$$T_{ad}(r, t) = T_s[c(t)] \exp\left[-\int_{c(t)}^{r} \frac{\alpha g}{C_P}\, dr'\right], \quad (9)$$

where the boundary condition that the temperature at the ICB ($r = c(t)$) is equal to the temperature of crystallization at this place $T_s[c(t)]$ has been used. To perform this integration, the ratio α/C_P was previously assumed to be constant (Labrosse et al., 2001), which may not be very accurate (Gubbins et al., 2003a) since, although C_P can be assumed constant, α varies with the radius through the identity:

$$\alpha = \frac{\gamma\rho C_P}{K_S}. \quad (10)$$

The Grüneisen parameter γ can also be assumed to be constant (Alfè et al., 2002b) and the isentropic incompressibility is to be obtained from the equation of state (4), by

$$K_S = \rho\left(\frac{\partial P}{\partial \rho}\right)_S = P + K_0\frac{\rho}{\rho_0}. \quad (11)$$

[1] Note, however, that Eq. (5) of (Labrosse et al., 2001) was mistyped: the last -1 was erroneously included in the square root.

This leads to

$$\alpha = \gamma C_P \frac{\rho_0}{K_0}\frac{1}{1 + \ln(\rho/\rho_0)}, \quad (12)$$

and a development at order 3 in radius is obtained using Eq. (6):

$$\alpha = \frac{\gamma C_P \rho_0}{K_0[\ln(\rho_c/\rho_0) + 1]}\left[1 + \frac{r^2}{L_\rho^2[\ln(\rho_c/\rho_0) + 1]}\right] \quad (13)$$

Using Eq. (13) and values given in Table 1, α is found to vary from $\alpha_c = (1.25 \pm 0.4) \times 10^{-5}\,\text{K}^{-1}$ at the center to $\alpha_{CMB} = (1.7 \pm 0.5) \times 10^{-5}\text{K}^{-1}$ at the CMB. This variation is important and has to be taken into account each time the temperature gradient is involved, like in the computation of the entropy production due to conduction along the adiabat (see below).

On the other hand, since we stop our developments to the third-order in r/L_ρ, αg is required only up to second-order to integrate the temperature profile

Table 1
Parameter values

Parameter	Value
Core radius[a], b (km)	3480 ± 5
Present inner-core radius[a], c_f (km)	1221 ± 1
Density at the center[b], ρ_c (Kg m^{-3})	$12.5 \pm 0.55 \times 10^3$
Density jump at ICB[c], $\Delta\rho$ (kg m^{-3})	500 ± 100
Density at 0 pressure[d], ρ_0 (kg m^{-3})	$7.5 \pm 0.1 \times 10^3$
Density length scale[e], L_ρ (km)	7400 ± 150
Thermal expansion coefficient at the center[e] α_c (K^{-1})	$1.3 \pm 0.1 \times 10^{-5}$
Specific heat[f] C_p (J kg^{-1} K^{-1})	850 ± 80
Entropy of crystallization[g], ΔS (J kg^{-1} K^{-1})	118 ± 12
Present temperature at ICB[h], $T_s(c_f)$ (K)	5600 ± 500
Grüneisen parameter[i], γ	1.5 ± 0.2
Thermal conductivity[j], k	50 ± 10

[a] From PREM Dziewonski and Anderson (1981) with a reasonable estimate for CMB and ICB topography.
[b] From PREM with 5% uncertainty Bolt (1991) and after subtraction of the density jump at the ICB.
[c] From PREM after subtraction of up to 1.7% density change upon freezing (Poirier and Shankland, 1993; Laio et al., 2000).
[d] About 5% depression assumed from pure iron.
[e] Derived from PREM (see text).
[f] From Stacey (1993) with 10% uncertainty.
[g] From Poirier and Shankland (1993).
[h] From Alfè et al. (2002a).
[i] From Alfè et al. (2002b).
[j] From Stacey and Anderson (2001).

according to Eq. (9), which means that the radius dependence of α will not appear in this profile, its value at the center being the relevant one. This partly justifies the assumption of a constant α made in the previous studies (Labrosse et al., 1997; Labrosse and Macouin, 2003) as was observed from numerical estimates by Gubbins et al. (2003a). The resulting adiabatic temperature profile is

$$T_{\mathrm{ad}}(r,t) = T_{\mathrm{s}}[c(t)] \exp\left[\frac{c^2(t)-r^2}{L_{\mathrm{T}}^2}\right],$$
$$\text{with } L_{\mathrm{T}} = \sqrt{\frac{3C_p}{2\pi\alpha_{\mathrm{c}}\rho_{\mathrm{c}}G}} = 6042 \pm 1400\,\mathrm{km}. \tag{14}$$

The solidification temperature T_{s} can be expressed in a similar form by use of the Lindemann law of melting, giving (Labrosse et al., 2001):

$$T_{\mathrm{s}}(r) = T_{s0} \exp\left[-2\left(1-\frac{1}{3\gamma}\right)\frac{r^2}{L_{\mathrm{T}}^2}\right]. \tag{15}$$

Before the inner-core crystallization, the temperature at the geocenter T_{c} is used as a parameter for the adiabat, giving

$$T_{\mathrm{ad}}(r,t) = T_{\mathrm{c}} \exp\left[\frac{-r^2}{L_{\mathrm{T}}^2}\right]. \tag{16}$$

The composition of the fluid outer core of the Earth is not known with precision (Poirier, 1994) but can be modeled as a binary mixture of iron and an unspecified light element of mass fraction ξ. It is likely that several light elements combine to produce the observed density deficit of the core compared to pure iron in the same condition of pressure and temperature but taking into account compositions that are more complicated than the binary mixture seems premature, considering the lack of constraints on this composition. Even though the composition of the core may have an important influence on some parameters that are crucial to the problem, like the compositional density jump at the inner-core boundary (ICB) (Alfè et al., 2002a), all these effects can be taken into account in the framework of this model, as uncertainties in these parameters. Different values used here are given in Table 1.

The fluctuations around this profiles average to zero in most terms relevant to the thermal evolution, that is everywhere except when their correlations with the velocity field are involved.

2.2. Energy balance

The energy balance of the present core (Gubbins, 1977; Gubbins et al., 1979; Buffett et al., 1992; Braginsky and Roberts, 1995; Lister and Buffett, 1995; Buffett et al., 1996; Labrosse et al., 1997, 2001),

$$Q_{\mathrm{CMB}} = Q_{\mathrm{ICB}} + Q_{\mathrm{C}} + Q_{\mathrm{L}} + E_{\chi} + Q_{\mathrm{R}} \tag{17}$$

expresses that the total heat loss of the core, the heat flow at the core mantle boundary, Q_{CMB}, is balanced by the sum of the energy sources: the heat flow coming from the inner-core, Q_{ICB}, the secular cooling, Q_{c}, the latent heat, Q_{L}, the compositional energy, E_{χ} (usually called gravitational energy, as discussed below) and the radiogenic heat, Q_{R}.

As explained in the following, all these source terms, save Q_{R} and Q_{ICB}, can be expressed as

$$Q_{\mathrm{X}} = P_{\mathrm{X}}(c)\frac{\mathrm{d}c}{\mathrm{d}t}, \tag{18}$$

which renders the integration of the growth of the inner-core radius ($r = c(t)$) straightforward.

The case of the latent heat is obvious since it is directly proportional to the rate of volume increase of the inner core, which gives:

$$P_{\mathrm{L}}(c) = 4\pi c^2 \rho(c) T_{\mathrm{s}}(c) \Delta S \tag{19}$$

with ΔS the entropy of fusion.

The secular cooling of the core is simply

$$Q_{\mathrm{c}} = -\int_V \rho C_P \frac{\partial T_{\mathrm{ad}}}{\partial t}\,\mathrm{d}V, \tag{20}$$

which can be computed directly from Eq. (9) and, since the adiabat is anchored to the crystallization temperature at the inner-core radius, will clearly be related to the growth rate of the inner core. The volume V on which the integration is performed needs however to be discussed. If, as in Eq. (17), the heat flow at the ICB is computed separately, this volume must be understood as the volume of the outer core only. The heat flow at ICB must then be obtained from the energy balance for the inner core which is however such a small fraction of the total volume of the core, that its contribution to the total balance is small (Labrosse et al., 1997). The cooling of the inner core can reasonably well be approximated by assuming that it is adiabatic (Labrosse et al., 2001), in which case, it is added to the cooling of the outer core and V is understood as the total volume of the core. In the case where radioactive elements are present in the inner core, their contribution to the heat flow at the ICB must not be forgotten and can be added to the radiogenic heat in the outer core, giving a total of $Q_R = h(t)M_N$, M_N being the mass of the core and h the rate of heat produced per unit mass which is supposed to be uniform in the whole core, and in particular not fractionating across the ICB.

Then, assuming the inner core to be adiabatic, the secular cooling of the whole core can be obtained from integration of Eq. (20) using Eqs. (14) and (15) for the temperature profiles and the resulting P_c function is:

$$P_c = 4\pi H^3 \rho_c C_P T_{s0} \left(1 - \frac{2}{3\gamma}\right) \frac{c}{L_T^2} \exp\left[\left(\frac{2}{3\gamma} - 1\right)\frac{c^2}{L_T^2}\right] I(H, b) \tag{21}$$

H being a length scale combining L_ρ and L_T

$$\frac{1}{H^2} = \frac{1}{L_\rho^2} + \frac{1}{L_T^2} \tag{22}$$

and I a function coming from the integration of $r^2 \exp(-r^2/H^2)$:

$$I(H, b) = \frac{\sqrt{\pi}}{2} \operatorname{erf}\frac{b}{H} - \frac{b}{H}\exp\left(-\frac{b^2}{H^2}\right), \tag{23}$$

with b the radius of the CMB.

The compositional energy comes from the redistribution of the light elements released at the ICB in the outer core which has a spatially varying chemical potential μ (Braginsky and Roberts, 1995):

$$E_\chi = \int_V \rho\dot{\xi}(\mu - \mu_{ICB})\,dV. \tag{24}$$

This energy is not equal to the total change of gravitational energy of the system, which is also caused by cooling of the Earth, and crystallizing the inner core. If the coefficient of chemical expansion α_ξ is uniform, the compositional energy can be proved to be equal to the change of gravitational energy due to redistribution of light elements only (Braginsky and Roberts, 1995). This is proportional to the rate at which light elements are released at the ICB, hence, to the growth rate of the inner core. We can then write $E_\chi = E_G = P_G(c)\,dc/dt$ of which a version

Table 2
Total energies

Energy	Value
Gravitational, $\int_0^{c_f} P_G(c)\,dc$	4.1 ± 1.0
Latent, $\int_0^{c_f} P_L(c)\,dc$	7.0 ± 2.0
Cooling, $\int_0^{c_f} P_c(c)\,dc$	18.2 ± 15.4
Total, $E_{tot} = \int_0^{c_f} (P_G(c) + P_L(c) + P_c(c))\,dc$	29.3 ± 18.8

Computed total energies and uncertainties in units of 10^{28} J.

integrated over the growth of the inner core was computed, within the framework of the present model, by Labrosse et al. (2001). The resulting leading order expression for P_G is:[2]

$$P_G(c) = \frac{8\pi^2}{3} G \Delta\rho \rho_c c^2 b^2 \left(\frac{3}{5} - \frac{c^2}{b^2}\right). \tag{25}$$

Finally, the energy balance can be written as

$$Q_{CMB}(t) = [P_c(c) + P_L(c) + P_G(c)]\frac{dc}{dt} + h(t)M_N, \tag{26}$$

and can be used to compute the inner-core growth history for any given heat flow history and concentration in radioactive elements. This equation can also be integrated between the onset of inner-core crystallization ($t = -a_{IC}$, a_{IC} being the age of the inner core) and the present time to get an equation for the age of the inner core (Labrosse et al., 2001). The values for the integrated energies are given in Table 2 and it can be seen that the secular cooling term is much larger than the one given in Labrosse et al. (2001), although error bars do overlap. This comes from different choices in the parameter values, mainly for α which, here, is computed from PREM. The values given here are more self-consistent and should be used in place of the previous ones.

Before the inner-core crystallisation, the balance is simply between the heat flow at the CMB and the sum of secular cooling and radiogenic heat

$$Q_{CMB} = Q_c + h(t)M_N, \tag{27}$$

where, now, the secular cooling is obtained by time derivation of the adiabat before the inner-core

[2] A higher order expression is used in the actual calculations below and is obtained from differentiating eq. 44 of Labrosse et al. (2001) with respect to the radius of the inner core.

Eq. (16):

$$Q_c = -2\pi C_P \rho_c H^3 I(H, b) \frac{dT_c}{dt}. \tag{28}$$

The energy balance (27) is used to compute the time evolution of the temperature of the core for the period preceding the appearance of the inner core.

2.3. Entropy balance

Ohmic and viscous dissipation do not appear in the global energy balance of a convective system because they are equilibrated internally by the work of buoyancy forces (Hewitt et al., 1975; Backus, 1975). To relate these quantities to the energy available to drive convection we need to derive an entropy balance for the core, which reads (e.g. Gubbins, 1977; Gubbins et al., 1979; Braginsky and Roberts, 1995; Roberts et al., 2003; Lister, 2003; Labrosse and Macouin, 2003):

$$\frac{Q_{CMB}}{T_{CMB}} = \frac{Q_{ICB}+Q_L}{T_{ICB}} + \int_V k\left(\frac{\nabla T}{T}\right)^2 dV + \int_V \frac{\phi}{T} dV + \int_V \frac{\rho}{T}\left(h - C_P\frac{\partial T_{ad}}{\partial t}\right) dV, \tag{29}$$

where each heat source appears divided by the temperature at which it is supplied and the volume V of integration is that of the outer core. The poorly constrained heat of reaction has not been included here and it can be argued to play a rather minor role (Gubbins et al., 2003b; Lister, 2003). Even if a turbulent viscosity is assumed, the viscous dissipation can be neglected in the core (Braginsky and Roberts, 1995) and the local dissipative heating is dominated by its Ohmic contribution,

$$\phi = \frac{J^2}{\sigma}, \tag{30}$$

with J the electric current density and σ the electrical conductivity. The gravitational energy can then be reintroduced in the problem by use of the energy Eq. (17) to replace Q_{CMB}:

$$\int_V \frac{\phi}{T} dV + \int_V k\left(\frac{\nabla T}{T}\right)^2 dV = \int_V \rho\left(h - C_P\frac{\partial T_{ad}}{\partial t}\right)\left(\frac{1}{T_{CMB}} - \frac{1}{T}\right) dV + (Q_{ICB}+Q_L)\left(\frac{1}{T_{CMB}} - \frac{1}{T_{ICB}}\right) + \frac{E_G}{T_{CMB}}. \tag{31}$$

In these equations, the Ohmic dissipation in the inner core and chemical diffusion have been neglected (Lister, 2003).

Several implications of this equation need to be pointed out. First, each thermal source for the dynamo, X say, that is provided to the core at the temperature T_X, is multiplied by a factor of the form $(1/T_{CMB}) - (1/T_X)$ which is reminiscent of the classical Carnot efficiency of heat engines. On the other hand, the gravitational energy due to the inner core chemical differentiation enters in the entropy Eq. (31) with a much larger factor of $1/T_{CMB}$, as was originally proposed in Braginsky (1964). The secular cooling and radioactive heating, that are mathematically equivalent (Krishnamurti, 1968), enter with a small efficiency because these sources are distributed over the whole core. Of course, these efficiency arguments are only indicative and the exact part of each heat source to the process of the geodynamo depends on the total energy that each source provide as well.

Another point to make is that all the energy sources on the right hand side of Eq. (31) are not only used to produce the magnetic field that appears as Ohmic heating in the dissipation term but also to maintain the conduction along the adiabatic temperature gradient. This second term on the left hand side can be computed by use of Eq. (3) to give

$$\int_V k\left(\frac{\nabla T}{T}\right)^2 dV = 4\pi\int_c^b k\left(\frac{\alpha g}{C_P}\right)^2 r^2 dr = 195\,\text{MW K}^{-1}. \tag{32}$$

To compute this quantity, it is important to get back to the definition of the adiabatic gradient Eq. (3), giving the right hand side in Eq. (32), and then to develop α and g according to Eqs. (13) and (7), respectively. One could be tempted to get the temperature gradient by deriving Eq. (14) but this would be inconsistent with the third-order development adopted here, since it comes itself from an integration of Eq. (3).

The question that we want to answer now is: what are the requirements for the core to be able to sustain a magnetic field of a given strength? By reasonably assuming the viscous part of the dissipation negligible, one could estimate the total Ohmic dissipation for this given magnetic field and use the entropy Eq. (31) to constrain the energy sources that are necessary to sustain it. All source terms of the Eq. (31), except the one arising from the radioactive heating, can be expressed as a function S_c of the inner-core radius times the inner-core growth rate:

$$\int_V \frac{\phi}{T}\,\mathrm{d}V + \int_V k\left(\frac{\nabla T}{T}\right)^2 \mathrm{d}V = S_c(c)\frac{\mathrm{d}c}{\mathrm{d}t} + \int_V \rho h\left(\frac{1}{T_{\mathrm{CMB}}} - \frac{1}{T}\right)\mathrm{d}V. \qquad (33)$$

This growth rate can then be inferred from the entropy requirements for any given radioactive heating rate. This can then be injected in the energy Eq. (26) to get the heat flux at the CMB necessary to produce this magnetic field. The question of the link between the observed magnetic field and the Ohmic dissipation is however far from obvious and will be addressed in the next section.

Before the existence of the inner core, the entropy balance is a simpler version of Eq. (31), involving only radiogenic heating and secular cooling on the right hand side. The secular cooling contribution for that period is parameterized by the temperature at the center and proportional to its time derivative so that the equivalent of Eq. (33) is now

$$\int_V \frac{\phi}{T}\,\mathrm{d}V + \int_V k\left(\frac{\nabla T}{T}\right)^2 \mathrm{d}V = S_T(T_c)\frac{\mathrm{d}T_c}{\mathrm{d}t} + \int_V \rho h\left(\frac{1}{T_{\mathrm{CMB}}} - \frac{1}{T}\right)\mathrm{d}V. \qquad (34)$$

The computation of the different contributions to S_c as well as S_T is rather obvious from their definition and the reference state (Section 2.1) and need not be described in detail here. One must be careful however not to include the inner core in the computation of the contribution from the secular cooling and radioactive heating.

3. Magnetic field scaling

It was shown, in the previous section, that if the Ohmic dissipation in the core is known, one can infer the heat flow at the CMB needed to maintain it. Compared to the approach used by Labrosse et al. (1997), we can then replace the heat flow at the CMB as control parameter by the Ohmic dissipation, which offers a hope of using magnetic measurements to constrain the thermal evolution of the Earth. The task is however not easier than before because the Ohmic dissipation in the core is not better known than the heat flow at the CMB. The reason for this will be discussed shortly here and some scaling options of the magnetic field will be presented.

The computation of the Ohmic dissipation in the core requires the knowledge of the electric current density (see Eq. (30)) which should be obtained from

$$J = \frac{\nabla \times B}{\mu_0}, \qquad (35)$$

μ_0 being the magnetic permeability. The difficulty lies in the limited knowledge we have about the magnetic field in the core, restricted to the large scale poloidal field. Eq. (35) implies that small scales of the field may have an important contribution to the total dissipation: a field B varying on a length scale l will contribute to the local dissipation as $B^2/(\mu_0 l)^2\sigma$. The important ingredient in computing the total Ohmic dissipation in the core is then the shape of the function $B(l)$ (the spectrum of the field) rather than the amplitude of the dipole.

Our knowledge of the magnetic field of the Earth is constantly increasing, thanks to the more systematic use of satellites (Hulot et al., 2002), but is still limited to the large scale poloidal field. The small scale (degree and order larger than around 15 (Hulot et al., 2002)) poloidal field in the core is screened from our measurements by the crustal magnetic field. The toroidal field is restricted to electrically conducting regions, which, to a very good approximation, excludes the mantle, and is therefore not observed at the surface. Assessing the contribution of these non-observed parts of the field to the total Ohmic dissipation in the core relies on some extrapolation of the magnetic field spectrum beyond the observable range and is model dependent. Roberts et al. (2003) discussed in details several possible choices of model and proposed a total dissipation $\Phi = \int \phi \mathrm{d}V$ between 1 TW and 2 TW. It is important to note that the largest contribution to that number comes from the non-observed part of the field, essentially the toroidal intermediate scale field. Using the dynamo model of Kuang and Bloxham (1997), Buffett (2002) proposed $0.1\mathrm{TW} \le \Phi \le 0.5\mathrm{TW}$, although without excluding the possibility of larger values.

The relevant parameter for the cooling of the core is not the total Ohmic dissipation but the total contribution S_Φ of Ohmic dissipation to the entropy budget, which can be related to Φ by the introduction of the typical temperature at which dissipation occurs, T_D, by (Roberts et al., 2003)

$$S_\Phi \equiv \int_V \frac{\phi}{T}\,dV = \frac{\Phi}{T_D}. \quad (36)$$

T_D is unknown but of course $T_{ICB} > T_D > T_{CMB}$. Any estimate in the total Ohmic heating in the core can then provide an estimate of the contribution of Ohmic dissipation to the entropy equation, with an additional uncertainty. This uncertainty is the same as the one involved in going from thermal energies provided to the core at its boundaries to the thermal buoyancy flux available for the dynamo (Lister and Buffett, 1995; Lister, 2003) which is also influenced by the distribution of Ohmic dissipation. The estimate given by Roberts et al. (2003), using the bounds on T_D, give $200\,\mathrm{MW\,K^{-1}} \leq S_\Phi \leq 600\,\mathrm{MW\,K^{-1}}$ whereas Gubbins et al. (2003a) estimate that the entropy production due to Ohmic heating is between $500\,\mathrm{MWK^{-1}}$ and $800\,\mathrm{MW\,K^{-1}}$. In the present study, $350\,\mathrm{MW\,K^{-1}} \leq S_\Phi \leq 700\,\mathrm{MW\,K^{-1}}$ is assumed. This is significantly larger than those of Buffett (2002) and the problems he encountered to maintain his largest value of Ohmic dissipation will then be emphasized here and solutions will be proposed.

Despite all these difficulties, one can always write that the total Ohmic dissipation in the core is

$$\Phi = \left(\frac{B_D^2}{\mu_0^2 \sigma l_D^2}\right) V, \quad (37)$$

B_D and l_D being typical values for the magnetic field and the length scale at which dissipation occurs. Of course, the link between these scales and the observed field and the size of the core are far from obvious but several assumptions can be done. One can for example assume that the shape of the spectrum $B(l)$ is not affected by a change in energy input, only its level being modified. In this case, the total dissipation will scale directly as the square of the intensity of the dipole (Stevenson et al., 1983; Labrosse and Macouin, 2003). The other extreme would be to assume that when the energy input is increased, the scale at which dissipation occurs decreases so that the intensity of the dipole is kept constant. This last scenario was defended by Stevenson (1984) who argued that a regime keeping the Elsasser number close to 1 is favored for the Earth's core. However, if the former assumption holds, the typical change in the intensity of the dipole in a thermal history model is only twofold (Labrosse and Macouin, 2003), implying a factor of 4 change in the Elsasser number, which may not be enough to make the system change regime, owing to the uncertainties inherent to this type of scale analysis. To summarize, the link between the total dissipation in the core and intensity of the dipole field is highly uncertain and variations of the dipole implied in the change of scenario are expected to be much smaller than the variations observed on very short time scales (Labrosse and Macouin, 2003).

4. Thermal and magnetic evolution models

4.1. General procedure

In all cases, the value of S_Φ at the present time ($t = 0$) and just before the onset of inner-core crystallization ($t = -a_{IC}$) are chosen as input parameters, and the relevant entropy and energy equations are then used to get the corresponding values for the heat flow at the CMB. This is possible since the temperature profiles are known at both instants. To compute the whole thermal evolution, the heat flow at each time is necessary and an interpolation (whose form is discussed below) between the two moments at which it is known is performed. Some fiddling is necessary because, although the values are known, the time at which the early one applies ($t = -a_{IC}$) is not known a priori. An iterative procedure is used: The inner core is first assumed to appear 1 Ga ago and a heat flow history is then obtained between this time and the present by interpolation. This, in turn, can be used to get the age of the inner core (Labrosse et al., 2001). This age is generally not equal to the initially assumed one and the same procedure has to be reiterated until convergence.

A heat flow exponentially decreasing with time can be defended on the base of parameterized thermal evolution models for the mantle (see discussion in Labrosse et al., 2001) due to the fact that the heat flow at the bottom of an internally heated convection system is mainly controlled by the arrival of cold down welling currents (Labrosse, 2002) and is then expected to decrease following the decrease of heat sources in the mantle. In order to get results as general as possible, a linear interpolation is also tried (and actually always used when the heat flow is found to increase with time), and gives very similar results in terms of the recent evolution (including the age of the inner core) and of course large differences when going backward to the origin of the system. The results considering the age of the inner core can then be considered as robust, within the framework of a monotonic variation. Such monotonicity is not meant to represent the actual time variations of the heat flow at the CMB but rather a running average of it, removing the variations associated with mantle dynamics while keeping those associated with thermal evolution. The time scale of mantle convection being long compared to core processes, the validity of such an averaging procedure is questionable but discussion of that point is postponed until Section 5.

Once the heat flow at the CMB is obtained for the present time and at the onset of inner core crystallization, the heat flow variation with time can also be extrapolated backward in time up to an age of the 4.5 Ga. This heat flow history can then be used to compute the whole thermal evolution of the core, using the equation of conservation of energy, and the corresponding evolution of the Ohmic dissipation, using the entropy equation.

The two periods of time defined by the appearance of the inner core (which is then computed before anything else) must be treated separately: At any time for the period before the existence of the inner core, the temperature at the center is simply obtained by analytical integration of the energy Eq. (27) between the appearance time of the inner core ($t = -a_{\mathrm{IC}}$, $T_{\mathrm{c}} = T_{s0}$) and the considered time ($t \le -a_{\mathrm{IC}}$):

$$T_{\mathrm{c}}(t) = T_{s0} - \frac{1}{2\pi C_P \rho_{\mathrm{c}} H^3 I(H, b)} \times \int_{-a_{\mathrm{IC}}}^{t} [Q_{\mathrm{CMB}}(t) - h(t) M_{\mathrm{N}}]\, \mathrm{d}t. \quad (38)$$

Of course, the energy Eq. (27) is then also used to get the time derivative of the temperature at the center which in turn can be used to get the Ohmic dissipation at that time. The time evolution of the system when the inner core is present is computed numerically (Runge–Kutta) using the energy conservation Eq. (26)

In addition to the values of S_{Φ} at present and just before the onset of inner-core crystallization, another parameter has to be chosen: the concentration in radioactive elements. Only ^{40}K is considered here for simplicity but including other elements would represent no difficulty (Labrosse et al., 2001).

4.2. Constant Ohmic dissipation

In the present section, the same dissipation as assumed for the present is maintained just before the onset of inner-core crystallization. The results of this case are summarized in Table 3 and will be explained now. Note that these results are a little different from those given in Labrosse and Macouin (2003) because, in that paper, the calculations were parameterized by the Ohmic dissipation 1 Ga ago, instead of just before the onset of the inner-core crystallization.

For the range of values chosen here for S_{Φ}, a heat flow at the CMB between roughly 7 and 12 TW is necessary. This value is very similar to the ones proposed

Table 3
Results for constant dissipation

$S_{\Phi}(t=0)$ (MW K^{-1})	350	500	700
$Q_{\mathrm{CMB}}(t=0)$ (TW)	7.4	9.5	12.3
$Q_{\mathrm{CMB}}(t=-a_{\mathrm{IC}})$ (TW)	15.6	19.8	25.5
a_{IC} (Ma)	843 (807)	660 (632)	512 (491)
$T_{\mathrm{CMB}}(t=-4.5\,\mathrm{Ga})$ (K)	1.1×10^4 (6385)	2.5×10^4 (7596)	9.3×10^5 (9604)

Results of the efficiency calculation in the cases where the same Ohmic dissipation is maintained just before the onset of inner-core crystallization as at present. Given are the heat flow at the CMB for the present time ($t = 0$) as well as just before the appearance of the inner core ($t = -a_{\mathrm{IC}}$), the age of the inner core and the temperature at the CMB just after core formation ($t = -4.5$ Ga). Three different values of the total dissipation have been considered and for each of these, the inter/extrapolation of heat flow is either exponential or linear (values in parenthesis). See text for more details.

by Roberts et al. (2003) and Gubbins et al. (2003b), showing that this result is not very sensitive to the differences in our respective models. Another point of comparison is given by the heat flow down the adiabatic temperature gradient at the top of the core, which is 7 ± 3TW with the parameters used in the present study. This value is quite high, in part because of the radius dependence of the thermal expansion coefficient (Section 2.1). In all cases presented here, the heat flow at the CMB is larger than this value, although only slightly in the case with the lowest dissipation, indicating that the outer core is thermally unstable throughout. Other options, involving a stratified layer at the top of the core (Labrosse et al., 1997), are possible and this question should be addressed if the contribution of Ohmic dissipation to the entropy equation is assumed to be lower than in the present study, like for example the minimum value proposed by Buffett (2002). This type of solution would require a modification of the model (Labrosse et al., 1997) and is out of the scope of the present paper, which is why no value of S_Φ below 350 MW K^{-1} was investigated here.

If the same dissipation has to be maintained, by thermal convection alone, just before the inner core started to crystallize, a much larger heat flow is necessary (between 15 and 26 TW), because of the lower efficiency of that process compared to compositional convection. When these heat flow values are extrapolated exponentially backward in time, the thermal evolution of the core can be computed and the initial temperature at the CMB is ridiculously high and this problem is not solved by decreasing the entropy requirements by a factor of 2 as chown by Buffett (2002). Although more reasonable, the results obtained using a linear extrapolation are still difficult to accept in the case of a large dissipation.

Figure 1 shows the time evolution of S_Φ relative to the present value, corresponding to the different cases of Table 3. It can be seen that the recent evolution of the system is not very sensitive to the choice of interpolation but that the differences become important when looking at the evolution prior to the existence of the inner core (IC). Extrapolating exponentially based on two different values so close in time is clearly dangerous but, as discussed by Labrosse et al. (2001), an exponential decrease of the heat flow at the CMB is generally obtained in parameterized models for the cooling of the mantle. This type of evolution

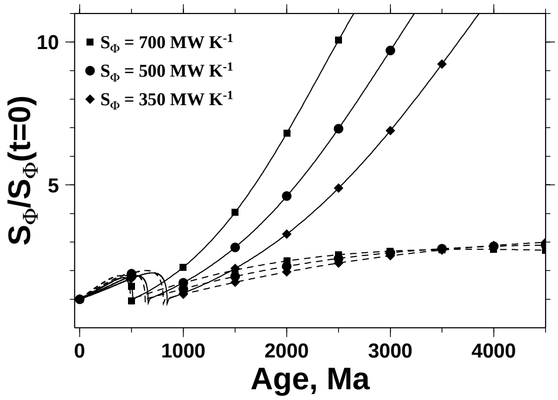

Fig. 1. Evolution of the contribution S_Φ of Ohmic dissipation to the entropy balance in the core relative to the present value when the same value is assumed just before the inner-core appearance. Different symbols are for different values of $S_\Phi(t = 0)$ (as labeled). Solid (dashed) lines are obtained when an exponential (linear) inter/extrapolation is assumed for the heat flow at the CMB (see text).

was for example obtained by Buffett (2002) who used the same type of thermodynamic approach as here, coupled with a parameterized evolution of the mantle.

One can note that this dissipation varies quite importantly with time, the present value being obtained only once before, that is just before the appearance of the inner core, as specified initially. In particular, there is a sharp increase of dissipation in the early history of the inner core, due to the qualitative change of the processes responsible for dynamo action. This shows that the assumption of a constant Ohmic dissipation throughout the Earth's history is not feasible since the sharp increase that has to result from inner core nucleation cannot be counter balanced by a change of heat flow at the CMB, on such a short time scale. This heat flow is anyway controlled by mantle processes that are largely independent of the dynamo and there is absolutely no reason to expect any such feedback.

If the Ohmic dissipation is assumed to scale as B^2 with the intensity of the dipole, an increase of this dipole of about 2×10^{22} Am^2 in a few tens on Ma is expected at the onset of inner core. Such a change is rather small compared to the variations observed in the intensity of the dipole in the last 300 Ma (Selkin and Tauxe, 2000; Labrosse and Macouin, 2003) which, obviously, cannot all be attributed to the appearance of the inner core. The appearance of the inner core seems then difficult to detect from that signal, until a better understanding of the link between the Ohmic dissipation in the core and the variations of the dipole have been reached.

4.3. Variable Ohmic dissipation

Since it was shown that the Ohmic dissipation in the core has to change rapidly when the inner core starts crystallizing, there is no reason to try and maintain it constant. It seems more reasonable to assume that S_Φ was a given factor smaller than at present. Labrosse and Macouin (2003) argued for a factor of 4, based on paleointensity data and the assumption that the dissipation scales as B^2 as a function of the intensity of the dipole. Assuming that S_Φ was four times lower just before the inner core started crystallizing ($t = -a_{\rm IC}$) than at present, a heat flow at that time essentially equal to the present one, or even smaller, is sufficient to maintain it (Table 4). This implies a reasonable initial temperature at the CMB.

The time evolution of the contribution of the Ohmic dissipation to the entropy equation relative to the present value is shown in Fig. 2. One can note that, except for the sharp increase due appearance of the inner core, it is rather constant, and actually much more so than when it was tried to keep it constant in Section 4.2. The maximum variation of the dipole field that would be obtained if it is assumed to scale as the square root of the dissipation would be again about 2×10^{22} Am^2, that is undetectable in the present database (Labrosse and Macouin, 2003).

Table 4
Results for variable dissipation

$S_\Phi (\equiv \int \phi/T\, dV)(t=0)(\mathrm{MW\,K^{-1}})$	350	500	700
$Q_{\rm CMB}(t=0)$ (TW)	7.4	9.5	12.3
vpace="0.5"$S_\Phi (\equiv \int \phi/T\, dV)$ ($t = -a_{\rm IC}$) (MW K − 1)	87.5	125	175
$Q_{CMB}(t=-a_{\rm IC})$ (TW)	8.1	9.2	10.6
vpace="0.5"$a_{\rm IC}$ (Ma)	1197	994	811
$T_{\rm CMB}(t=-4.5\,{\rm Ga})$ (K)	4784	4781	4695

Results of the efficiency calculations in the case where the contribution S_Φ of Ohmic dissipation to the entropy balance is assumed to be four times smaller just before the appearance of the IC than at present. An exponential interpolation of the heat flow has been assumed for the case with lowest S_Φ and a linear one for the others. A linear interpolation in the case of $S_\Phi = 350\,\mathrm{MW\,K^{-1}}$ gives almost identical results.

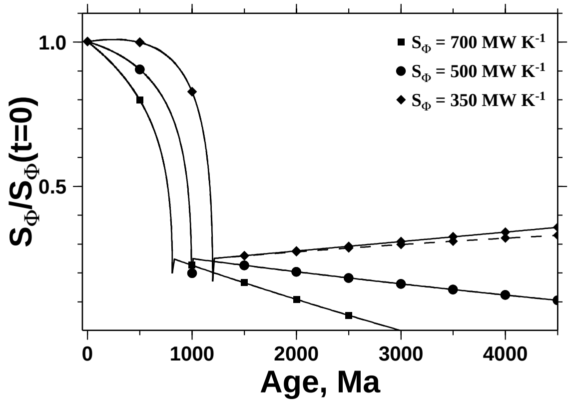

Fig. 2. Evolution of the Ohmic dissipation in the core relative to the present value when a value 4 times smaller is assumed just before the inner core appearance than at present. Different symbols are for different values of $S_\Phi(t=0)$ (as labeled). The case with the lowest S_Φ uses either exponential (plain line) or linear (dashed line) interpolation whereas the other two cases only use a linear interpolation (see text).

Both linear and exponential interpolation were used in the only case that has a decreasing heat flow at the CMB (a linear interpolation is always used in the case of an increasing heat flow), that is the case with the smallest dissipation. The results are almost identical, as shown in Fig. 2 and, compared to the results given in Table 4, only the initial temperature at the CMB would be slightly different (4777 K instead of 4784 K in the case of a linear interpolation).

It can be noted in Fig. 2 that the case with the greatest S_Φ has a dissipation going down to zero at about $t = -3$ Ga. This happen because the heat flow at the CMB, which decreases with age in that case, is such that its contribution to the entropy equation approaches the one due to conduction along the adiabatic gradient, leaving no room for Ohmic dissipation. In this scenario, the magnetic field would not have appeared before 3 Ga ago. Of course it relies on the choice of values for S_Φ both at present and just before the onset of IC crystallization. The factor of 4 between these values is reasonable (Labrosse and Macouin, 2003) but other values could be defended as well. This ratio could be adjusted in order to get a perfectly constant heat flow at the CMB or slightly decreasing one, producing a temporal evolution somewhat between these of Figs. 1 and 2.

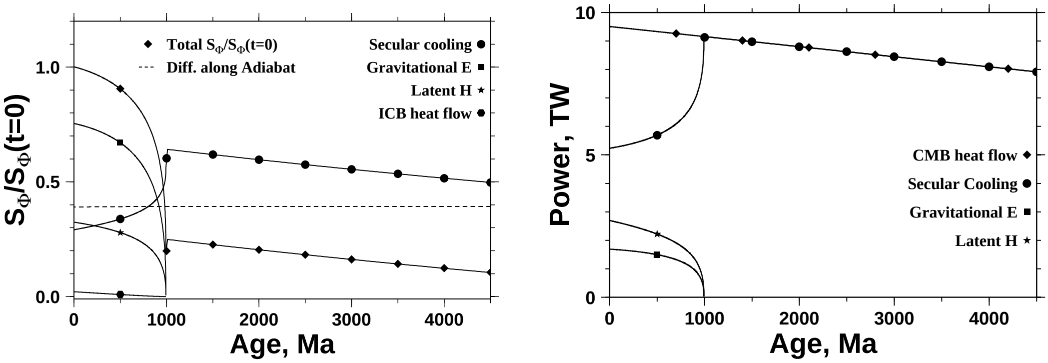

Fig. 3. Time evolution of the entropy (left) and energy (right) balances in the case of a present dissipation $S_\Phi = 500\,\mathrm{MW\,K^{-1}}$ and a value 4 times smaller just before the onset of inner-core crystallization. Each contribution to the entropy balance is normalized by the present value of S_Φ and the total heat flow down the adiabatic gradient at the top of the core (not shown) is fairly constant and equal to 7 TW.

Figure 3 shows the evolution of the entropy (left) and energy (right) balances in the system with $S_\Phi = 500\,\mathrm{MW\,K^{-1}}$. These clearly illustrate that, since the appearance of the inner core, the compositional energy, although rather small, plays the major part in maintaining the magnetic field. It also shows that the conduction along the adiabatic gradient in the core is an important contribution to the entropy balance, that can even be larger than the Ohmic dissipation before the existence of the inner core, as in the case considered here.

4.4. Effect of radioactive heating

Including radioactive heating in the core has been proposed as a heat source from the very beginning of the convective dynamo theory (e.g. Bullard, 1949) but the question of the concentration of radioactive elements in the Earth's core has never been settled. Including its effect into a thermodynamic theory of the Earth's core is then legitimate since it might help to understand the thermal evolution of core and also provide arguments for or against the existence such heat source in the core. The debate has mainly focused on ^{40}K (although other options could be considered, Labrosse et al., 2001) because it was originally proposed, from cosmochemical arguments, that a large part of the original supply to the forming Earth was stored in the core. The actual concentration of potassium in the core depends on its partition coefficient between silicates and iron at the condition under which the core formed. Reviewing the different experimental results addressing this question is clearly out of the scope of the present paper (see Roberts et al., 2003, for a discussion of the different arguments in the debate) but it is enough to say that the resulting concentrations in potassium in the core vary largely between being negligible (e.g. Oversby and Ringwood, 1972) up to 1500 ppm (Roberts et al., 2003), that would be producing about 10 TW at present. The most recent experimental results (Gessmann and Wood, 2002; Rama Murthy et al., 2003) argue for a concentration of potassium of O(100) ppm, producing less than 1 TW at present. Acknowledging the important sources of uncertainties in that parameter, as well as the possible implications in the case of other planets, different values for the concentration in potassium will be considered here, up to 750 ppm.

To keep the number of different variables small, S_Φ is assumed to be the same just before the onset of inner-core crystallization and at present, $S_\Phi(t = 0) = S_\Phi(t = -a_{\mathrm{IC}}) = 500\,\mathrm{MW\,K^{-1}}$, and the results are given in Table 5 where the corresponding results without radioactive heating have been replicated. It can be seen that the present value of the heat flow at the CMB necessary to maintain a given Ohmic dissipation, is larger than in the corresponding non-radioactive case. This comes from the lower efficiency of radioactive heating compared to other energy sources, due to it being released mostly at the top of the core. This is also true for the heat flow just before the onset of inner-core crystallization, but to a lesser

Table 5
Effect of radioactivity

K (ppm)	0	250	500	750
$Q_{\mathrm{CMB}}(t=0)$ (TW)	9.5	10.6	11.6	12.7
$M_{\mathrm{N}}h(t=0)$ (TW)	0	1.8	3.6	5.4
$Q_{\mathrm{CMB}}(t=-a_{\mathrm{IC}})$ (TW)	19.8	20.3	20.1	21.4
a_{IC} (Ma)	660 (632)	731 (732)	829 (795)	979 (934)
$M_{\mathrm{N}}h(t=-4.5\,\mathrm{Ga})$ (TW)	0	21.8	43.7	65.6
$Q_{\mathrm{CMB}}(t=-4.5\,\mathrm{Ga})$ (TW)	1429 (83)	585 (73)	273 (63)	140 (54)
$T_{\mathrm{CMB}}(t=-4.5\,\mathrm{Ga})$ (K)	2.5×10^4 (7596)	1.4×10^4 (6653)	9128 (5735)	6300 (4837)

Results of the thermal evolution calculation for cases with potassium in the core as function of its concentration K. In all these cases, $S_{\Phi}(t=0)=S_{\Phi}(t=-a_{\mathrm{IC}})=500\,\mathrm{MW\,K^{-1}}$. When they differ, results obtained with both an exponential and a linear (values in parenthesis) inter/extrapolation are displayed.

extent because the difference between the efficiency of radioactive heating and other thermal sources is less important than with compositional energy (see Eq. (31)). Having a larger heat flow at the CMB than in the non-radioactive case implies that the age of the inner-core is only a few hundreds of Ma larger than in the corresponding case without radioactivity. It is certainly not enough to make it as large as the age of the Earth itself. In order to clarify this point that may seem counter intuitive, let's consider the extreme (and non realistic) case where all radioactive elements are located at the top of the core, at a temperature equal to T_{CMB}. Eq. (33) shows that radioactivity in that case contributes nothing to maintaining the Ohmic dissipation. The growth rate of the inner core has to be kept the same as in the non-radioactive case to maintain the same dissipation and the resulting inner-core age is identical. On the other hand, radioactivity contributes completely to the energy balance (26) so that the heat flow at the CMB is increased by that amount. Now, in the actual case where the concentration in radioactive elements is uniformly distributed in the core (still mostly at the top in mass average), it contributes a little to the entropy Eq. (33), making the growth rate of the inner-core a little smaller and the inner core a little older. It still contributes completely to the energy balance (26), making the heat flow at the CMB larger than in the non-radioactive case.

On the other hand, the presence of radioactivity in the core helps the model in making the initial temperature more reasonable than in the corresponding non radioactive cases, and this for two reasons. First, because although the present heat flow at the CMB that is required is larger than in the non-radioactive case, the difference is less important for the time just before the onset of the inner-core crystallization which itself is slightly older, both factors making, via the extrapolation procedure, a smaller heat flow at the origin. Second, for the same initial heat flow, a large part of it is used to extract radiogenic heat instead of cooling down the core, which does not need to start as hot as in the non radioactive cases. In fact, the core may even start by heating up if the radioactive heating overcomes the heat flow at the CMB, as is the case in the calculation with the highest radioactivity and a heat flow linearly varying with time (Table 5).

This situation of an initially heating core can lead to a scenario with an inner core existing over the whole history of the Earth: If the core is formed with already a significant solid portion and a radioactive heating larger than the heat flow at the CMB, the inner core would start to melt and when the decay of radioactive heating makes it lower than the heat flow at the CMB, it would start to crystallize again. This scenario was proposed by Buffett (2003) and would be the only one able to satisfy the interpretation that the osmium signal seen in some samples is a signature of core mantle chemical interaction (Brandon et al., 1998; Meibom and Frei, 2002). This scenario is however difficult to accept since, in addition to a huge concentration in radioactive elements (enough to maintain the dynamo as the only heat source and melt the inner core), it requires a very cold start whereas the gravitational energy released by the formation of the core is sufficient to heat up the whole Earth by about 2000 K (Flasar and Birch, 1973).

5. Discussions and conclusion

Although a general agreement has been achieved in the community regarding the thermodynamics of the Earth's core, some debates persist in terms of parameter values and on the choice of the quantities that should be assumed constant in a thermal history calculation. These questions may seem rather secondary in importance but they need to be addressed if one wants to make predictions relevant to other fields of Earth sciences. If the Ohmic dissipation in the core is assumed to be the same before the existence of the inner core as at present, a much larger heat flow at the CMB is necessary then to maintain it, which may require a unacceptable initial temperature at the origin of the Earth. However, the heat flow at the core mantle boundary is a result from mantle dynamics and evolves on time scales much longer than those relevant to core dynamics. For this reason, the sharp increase in the efficiency of convection at the onset of the inner-core crystallization must express itself as a sharp increase in Ohmic dissipation. There is then no reason to keep this parameter as a constant in a thermal evolution model. If this assumption is relaxed and the dissipation before the inner core appeared is assumed to be smaller than at present by some factor, 4 say, about the same heat flow as at present is enough to maintain it, giving very reasonable initial temperature in the system. Of course there is no reason either to expect the heat flow at the CMB to be constant with time and intermediate solutions (with a smaller factor between the values of Ohmic dissipation at present and just before the onset of the inner core) are possible and would give reasonable initial temperatures.

Another parameter which can be adjusted is the concentration in radioactive elements, like ^{40}K considered here. Labrosse et al. (2001) showed that, if the heat flow at the CMB is kept the same as in the cases with no radioactive heating, an older inner core could be obtained. However, the relevant parameter to be kept identical if we want to compare models with and without radioactivity is the present Ohmic dissipation. Because radioactive heating is the least efficient of all heat sources, a much larger present heat flow at the CMB is required in presence of radioactivity to maintain the same dissipation than in the case with no radioactivity. This results in an inner core only a few hundreds of Ma older than in the non-radioactive core cases. This is in sharp contrast to what one gets when keeping the heat flow at the CMB constant while adding radioactive elements in the core without considering the magnetic field generation (Labrosse et al., 2001). On the other hand, the presence of radioactivity in the core would provide an energy source alternative to the secular cooling before the existence of the inner core, which would help to make a colder start than in the non-radioactive case.

If one wants to get a very old inner core, a much smaller Ohmic dissipation than the values taken here are necessary, as for example the minimum dynamo requirements proposed by Buffett (2002) which then produce a heat flow at the CMB of about 2 TW. This value is much smaller than the heat flow down the adiabatic gradient at the top of core, which would require the development of a thick stratified layer at the top of the core (Labrosse et al., 1997; Lister and Buffett, 1998). The present model needs to be modified to account for that possibility, since adiabaticity has been assumed throughout the core. Including this possibility might help since it would reduce the entropy produced by conduction along the average temperature profile. This contribution to the entropy balance is of the same order as the contribution from Ohmic dissipation. It can even be larger in some scenarios, like before the existence of the inner core in the case presented in Fig. 2. On the other hand, as pointed out by Gubbins et al. (2003a), reducing the temperature gradient also reduces the efficiency of all heat sources that involve the difference in temperature between the source and the CMB.

The only way of getting a inner core as old as the Earth while keeping the heat flow at the CMB within reasonable bounds consists in having a core originally formed with a large inner core that is then melted by a strong radioactive heating and then subsequently crystallized again (Buffett, 2003). This last solution is awkward since it requires, a large concentration in radioactive elements, a low initial heat flow at the CMB and very cold formation of the core, which is not favored by thermodynamics of core formation (Flasar and Birch, 1973).

An important development that should come in the future is a coupling of the present model for the thermal evolution of the core and a proper thermal history model for the mantle. Several such attempts have already been made (Stevenson et al., 1983;

Mollett, 1984; Labrosse, 1997; Yukutake, 2000; Grigné and Labrosse, 2001; Buffett, 2002; Nimmo et al., 2003) but the reliability of these models can be questioned for several reasons. First, most of these models assume a heat flux across the CMB following a $Ra^{1/3}$ scaling, Ra being the Rayleigh number defined with the local properties of the boundary layer. Such a scaling is typical of high Rayleigh number convection where the boundary layer has a dynamics essentially independent of the interior flow. Although this assumption is perfectly valid for the surface boundary layer, the dynamics of the bottom boundary layer of an internally heated system is found to be mostly controlled by the arrival and spreading of cold plumes (Labrosse, 2002). This effect is also increased by the temperature dependence of viscosity (Schaeffer and Manga, 2001) and the large scale plate flow (Jellinek et al., 2003). The proper scaling for the bottom boundary layer of the mantle is then largely unknown, even in the simple purely thermal case. This is the reason why the core is modeled separately in this paper, the heat flow history at the CMB being computed on the basis of Ohmic dissipation requirements.

Another shortcoming of coupled core and mantle models lies in the use of a quasi-static assumption, inherent to all parameterized models of mantle convection. The time during which the planet evolves in a transient that cannot be properly described by such laws depends on the dynamical regime that is considered and may be non negligible (e.g. Choblet and Sotin, 2000). Moreover, these parameterized models can only describe the evolution of the system averaged over a convective cycle. This type of averaging is perfectly justified when applied to the core, like in the model presented here, because of the smallness of temporal and lateral fluctuations around the mean state (e.g. Braginsky and Roberts, 1995) and because the secular evolution is very slow compared to the dynamics. On the other hand, fluctuations of heat fluxes at the boundaries of the convective mantle are likely to be large, both in time and space. This is for example the case in the much simplified case of a constant viscosity fluid presented by Sotin and Labrosse (1999) and it is even more so in the case of the more complex rheologies like those allowing the development of plate-like flows (Tackley, 2000) and the associated dominance of large scale flow. In this system, plate boundaries move in response to their mutual interaction, inducing important heat flux variations on very large time scales (Labrosse and Tackley, 2001). In the Earth, a similar behavior is likely to occur and induce heat flow variations on time scales of the order of 400 Ma, typical of the Wilson cycle. This time scale is not negligibly small compared to the age of the Earth and even less so when compared to the age of the inner core as found in the present study. Including variations on this time scale is certainly challenging but is the next step toward a better understanding of the thermal and magnetic evolution of the Earth's core.

Acknowledgements

Reviews by F. Nimmo and B. Buffett greatly helped to improve the present paper. Discussions with many colleagues, particularly D. Brito, P. Cardin, E. Dormy, D. Gubbins, R. Hide, G. Hulot, C. Jaupart, J.-L. Le Mouël, J. Lister, P. Roberts, G. Schubert, are greatly acknowledged. Figures were done using the generic mapping tool of Wessel and Smith (1998). This is IPGP contribution no. 1907.

References

Alfè, D., Gillan, M.J., Price, G.D., 2002a. Composition and temperature of the Earth's core constrained by combining ab initio calculations and seismic data. Earth Planet. Sci. Lett. 195, 91–98.

Alfè, D., Gillan, M.J., Price, G.D., 2002b. Iron under Earth's core conditions: liquid-state thermodynamics and high-pressure melting curve from ab initio calculations. Phys. Rev. B 65, 165118.

Backus, G.E., 1975. Gross thermodynamics of heat engines in deep interior of earth. Proc. Nat. Acad. Sci. U.S.A. 72, 1555–1558.

Bolt, B.A., 1991. The precision of density estimation deep in the Earth. Q. J. R. astr. Soc. 32, 367–388.

Braginsky, S.I., 1964. Magnetohydrodynamics of the Earth's core. Geomagn. Aeron. 4, 898–916.

Braginsky, S.I., Roberts, P.H., 1995. Equations governing convection in Earth's core and the geodynamo. Geophys. Astrophys. Fluid Dyn. 79, 1–97.

Brandon, A.D., Walker, R.J., Morgan, J.W., Norman, M.D., Prichard, H.M., 1998. Coupled ^{186}Os and ^{187}Os evidence for core–mantle interaction. Science 280, 1570–1573.

Buffett, B.A., 2002. Estimates of heat flow in the deep mantle based on the power requirements for the geodynamo. Geophys. Res. Lett. 29, 7.

Buffett, B.A., 2003. The thermal state of Earth's core. Science 299, 1675–1677.

Buffett, B.A., Huppert, H.E., Lister, J.R., Woods, A.W., 1992. Analytical model for solidification of the Earth's core. Nature 356, 329–331.

Buffett, B.A., Huppert, H.E., Lister, J.R., Woods, A.W., 1996. On the thermal evolution of the Earth's core. J. Geophys. Res. 101, 7989–8006.

Bullard, E.C., 1949. The magnetic field within the Earth. Proc. R. Soc. Lond. A 197, 433–453.

Choblet, G., Sotin, C., 2000. 3D thermal convection with variable viscosity: can transient cooling be described by a quasi-static scaling law? Phys. Earth Planet. Inter. 119, 321–336.

Dziewonski, A.M., Anderson, D.L., 1981. Preliminary reference Earth model. Phys. Earth Planet. Inter. 25, 297–356.

Flasar, F.M., Birch, F., 1973. Energetics of core formation: a correction. J. Geophys. Res. 78, 6101–6103.

Gessmann, C.K., Wood, B.J., 2002. Potassium in the Earth's core? Earth Planet. Sci. Lett. 200, 63–78.

Grigné, C., Labrosse, S., 2001. Effects of continents on Earth cooling: thermal blanketing and depletion in radioactive elements. Geophys. Res. Lett. 28, 2707–2710.

Gubbins, D., 1977. Energetics of the Earth's core. J. Geophys. 43, 453–464.

Gubbins, D., Alfè, D., Masters, G., Price, D., Gillan, M.J., 2003a. Can the Earth's dynamo run on heat alone? Geophys. J. Int., in press.

Gubbins, D., Alfè, D., Masters, G., Price, D., Gillan, M.J., 2003b. Gross thermodynamics of two-component core convection. Geophys. J. Int., submitted for publication.

Gubbins, D., Masters, T.G., Jacobs, J.A., 1979. Thermal evolution of the Earth's core. Geophys. J. R. Astr. Soc. 59, 57–99.

Hewitt, J.M., McKenzie, D.P., Weiss, N.O., 1975. Dissipative heating in convective flows. J. Fluid Mech. 68, 721–738.

Hulot, G., Eymin, C., Langlais, B., Mandea, M., Olsen, N., 2002. Small-scale structure of the geodynamo inferred from Oersted and Magsat satellite data. Nature 416, 620–623.

Jellinek, A.M., Gonnermann, H.M., Richards, M.A., 2003. Plume capture by divergent plate motions: Implications for the distribution of hotspots, geochemistry of mid-ocean ridge basalts and heat flux at the core–mantle boundary. Earth Planet. Sci. Lett. 205, 361–378.

Krishnamurti, R., 1968. Finite amplitude convection with changing mean temperature. Part 1. Theory. J. Fluid Mech. 33, 445–455.

Kuang, W., Bloxham, J., 1997. An Earth-like numerical dynamo model. Nature 389, 371–374.

Labrosse, S., 1997. Contribution à l'étude du refroidissement de la Terre. Ph.D. thesis, Université Paris-7, Denis Diderot.

Labrosse, S., 2002. Hotspots, mantle plumes and core heat loss. Earth Planet. Sci. Lett. 199, 147–156.

Labrosse, S., Macouin, M., 2003. The inner core and the geodynamo. C. R. Geosciences 335, 37–50.

Labrosse, S., Poirier, J.-P., Le Mouël, J.-L., 1997. On cooling of the Earth's core. Phys. Earth Planet. Inter. 99, 1–17.

Labrosse, S., Poirier, J.-P., Le Mouël, J.-L., 2001. The age of the inner core. Earth Planet. Sci. Lett. 190, 111–123.

Labrosse, S., Tackley, P.J., 2001. Heat transfer by mantle convection in a plate tectonic regime: 2D systematic investigation. Eos Trans. AGU, 82, Fall Meet. Suppl., Abstract U22A-0008.

Laio, A., Bernard, S., Chiarotti, G.L., Scandolo, S., Tosatti, E., 2000. Physics of iron at Earth's core conditions. Science 287, 1027–1030.

Lister, J.R., 2003. Expressions for the dissipation driven by convection in the Earth's core. Phys. Earth Planet. Inter., this issue.

Lister, J.R., Buffett, B.A., 1995. The strength and efficiency of the thermal and compositional convection in the geodynamo. Phys. Earth Planet. Inter. 91, 17–30.

Lister, J.R., Buffett, B.A., 1998. Stratification of the outer core at the core–mantle boundary. Phys. Earth Planet. Inter. 105, 5–19.

Meibom, A., Frei, R., 2002. Evidence for an ancient Osmium Isotopic Reservoir in Earth. Science 296, 516–518.

Mollett, S., 1984. Thermal and magnetic constraints on the cooling of the Earth. Geophys. J. R. Astr. Soc. 76, 653–666.

Nimmo, F., Price, G.D., Brodholt, J., Gubbins, D., 2003. The influence of potassium on core and geodynamo evolution. Geophys. J. Int. submitted for publication.

Oversby, V.M., Ringwood, A.E., 1972. Potassium distribution between metal and silicate and its bearing on the occurence of potassium in the Earth's core. Earth Planet. Sci. Lett. 14, 345–347.

Poirier, J.-P., 1994. Light elements in the Earth's core: a critical review. Phys. Earth Planet. Inter. 85, 319–337.

Poirier, J.-P., Shankland, T.J., 1993. Dislocation melting of iron and temperature of the inner core boundary, revisited. Geophys. J. Int. 115, 147–151.

Poirier, J.-P., Tarantola, A., 1998. A logarithmic equation of state. Phys. Earth Planet. Inter. 109, 1–8.

Rama Murthy, V., van Westrenen, W., Fei, Y., 2003. Radioactive heat sources in planetary cores: experimental evidence for potassium. Nature 423, 163–165.

Roberts, P.H., Jones, C.A., Calderwood, A.R., 2003. Energy fluxes and Ohmic dissipation in the Earth's core. In: Jones, C.A., Soward, A.M., Zhang, K. (Eds.), Earth's Core and Lower Mantle. Taylor, London.

Schaeffer, N., Manga, M., 2001. Interaction of rising and sinking mantle plumes. Geophys. Res. Lett. 28, 455–458.

Selkin, P.A., Tauxe, L., 2000. Long-term variations in palaeointensity. Phil. Trans. R. Soc. Lond. A 358, 1065–1088.

Sotin, C., Labrosse, S., 1999. Three-dimensional thermal convection of an isoviscous, infinite-Prandtl-number fluid heated from within and from below: applications to heat transfer in planetary mantles. Phys. Earth Planet. Inter. 112, 171–190.

Stacey, F.D., 1993. Thermodynamics relationships and the properties of iron at Earth's core conditions. In: Proceedings of the Conference for the Association for the International Advancement of High Pressure. Colorado Springs, Colorado.

Stacey, F.D., Anderson, O.L., 2001. Electrical and thermal conductivities of Fe–Ni–Si alloy under core conditions. Phys. Earth Planet. Inter. 124, 153–162.

Stevenson, D., 1984. The energy flux number and three types of planetary dynamo. Astronomische Nachrichten 305, 257–264.

Stevenson, D., Spohn, T., Schubert, G., 1983. Magnetism and thermal evolution of the terrestrial planets. Icarus 54, 466–489.
Tackley, P.J., 2000. Self-consistent generation of tectonic plates in time-dependent, three-dimensional mantle convection simulations. Part 1. Pseudoplastic yielding. Geochem. Geophys. Geosyst., 1, 2000Gc000041.
Wessel, P., Smith, W.H.F., 1998. New, improved version of generic mapping tools released. EOS Trans. Am. Geophys. U. 79, 579.
Yukutake, T., 2000. The inner core and the surface heat flow as clues to estimating the initial temperature of the Earth's core. Phys. Earth Planet. Inter. 121, 103–137.